1.

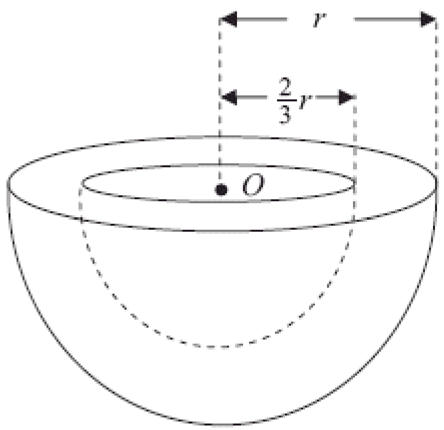


Figure 1

A bowl B consists of a uniform solid hemisphere, of radius r and centre O, from which is removed a solid hemisphere, of radius $\frac{2}{3}r$ and centre O, as shown in Figure 1.

(a) Show that the distance of the centre of mass of B from O is $\frac{65}{152}r$.

(5)

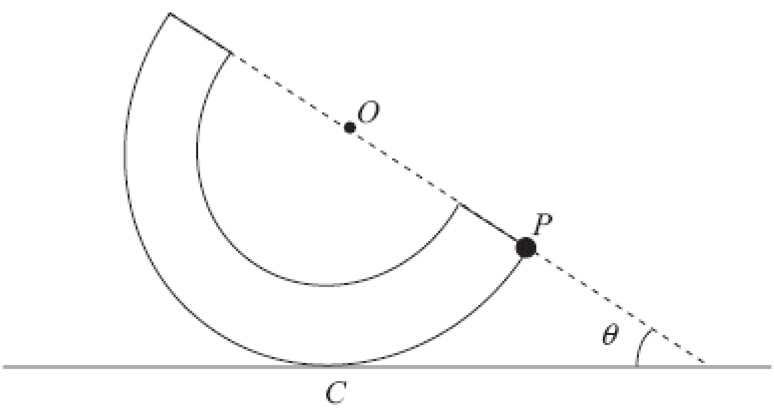


Figure 2

The bowl B has mass M. A particle of mass kM is attached to a point P on the outer rim of B. The system is placed with a point C on its outer curved surface in contact with a horizontal plane. The system is in equilibrium with P, O and C in the same vertical plane. The line OP makes an angle θ with the horizontal as shown in Figure 2. Given that $\tan\theta = \frac{4}{5}$,

(b) find the exact value of k.

(5)
(Total 10 marks)

2. A light elastic string has natural length 8 m and modulus of elasticity 80 N.

The ends of the string are attached to fixed points *P* and *Q* which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below *PQ*.

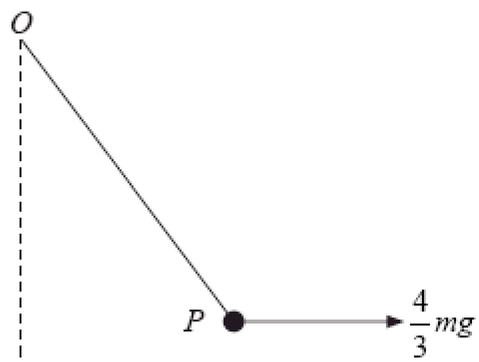
(a) Calculate the weight of the particle.

(6)

(b) Calculate the elastic energy in the string when the particle is in this position.

(3)
(Total 9 marks)

3.



A particle *P* of mass *m* is attached to one end of a light elastic string, of natural length a and modulus of elasticity 3*mg*. The other end of the string is attached to a fixed point *O*.

The particle *P* is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to *P*.

This force acts in the vertical plane containing the string, as shown in the diagram above. Find

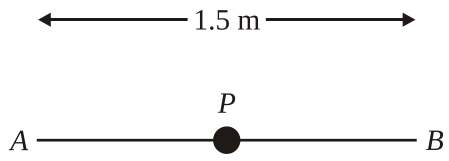
(a) the tension in the string,

(5)

(b) the elastic energy stored in the string.

(4)
(Total 9 marks)

4. Two light elastic strings each have natural length 0.75 m and modulus of elasticity 49 N. A particle P of mass 2 kg is attached to one end of each string. The other ends of the strings are attached to fixed points A and B, where AB is horizontal and AB = 1.5 m.



The particle is held at the mid-point of AB. The particle is released from rest, as shown in the figure above.

(a) Find the speed of P when it has fallen a distance of 1 m.

(6)

Given instead that P hangs in equilibrium vertically below the mid-point of AB, with $\angle APB = 2\alpha$,

(b) show that $\tan\alpha + 5\sin\alpha = 5$.

(6)
(Total 12 marks)

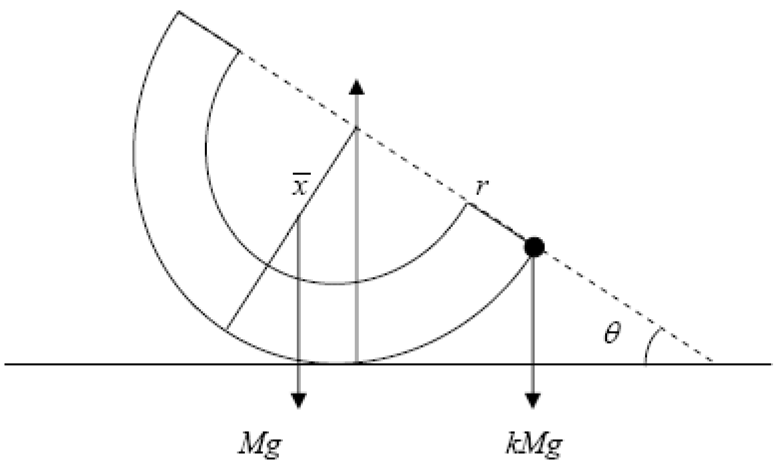
1. (a) Mass ratios $\begin{matrix} s & B & S \\ 8 & 19 & 27 \end{matrix}$ anything in correct ratio B1

$\bar{x} \quad \frac{3}{8}\times\frac{2}{3}r \quad \bar{x} \quad \frac{3}{8}r$ B1

$8\times\frac{1}{4}r+19\bar{x}=27\times\frac{3}{8}r$ M1 A1ft

$\bar{x}=\frac{65}{152}r$ * A1 5

(b)

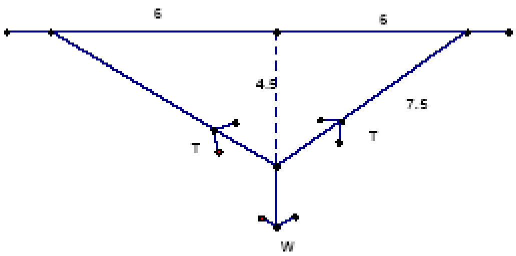


$Mg\times\ \bar{x}\sin\theta = kMg\times r\cos\theta$ M1 A1=A1

leading to $k=\frac{13}{38}$ M1 A1 5

[10]

2. (a)



Resolving vertically: $2T\cos\theta = W$ M1A2,1,0

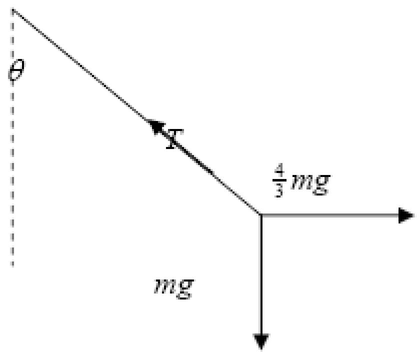
Hooke's Law: $T=\frac{80\times3.5}{4}$ M1A1

$W = 84\text{N}$ A1

(b) EPE = $2\times\frac{80\times3.5^2}{2\times4}$, =245 (or awrt 245) M1A1ft,A1 (alternative $\frac{80 \times 7^2}{16} = 245$)

[9]

3. (a)



$(\leftarrow)$	$T\sin\theta = \frac{4}{3mg}$	M1 A1	
$(\uparrow)$	$T\cos\theta = mg$	A1	
	$T^2 = \left(\frac{4}{3}mg\right)^2 + (mg)^2$	M1	
Leading to	$T = \frac{5}{3}mg$	A1	5

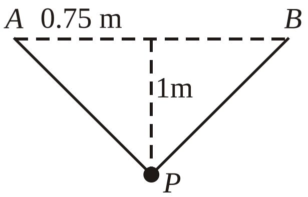
(b) HL $T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3}mg = \frac{3mge}{a}$ ft their T M1 A1ft

$e = \frac{5}{9}a$

$E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga$ M1 A1 4

[9]

4. (a)



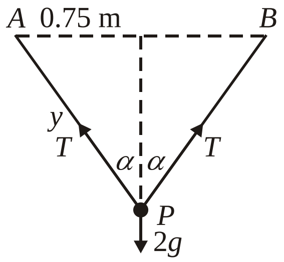
$AP = \sqrt{(0.75^2 + 1^2)} = 1.25$ M1 A1

Conservation of energy

$\frac{1}{2}\times 2\times v^2 + 2\times\frac{49\times 0.5^2}{2\times 0.75} = 2g\times 1$ −1 for each incorrect term M1 A2 (1, 0)

Leading to $v \approx 1.8$ (ms^{-1}) accept 1.81 A1 6

(b)



R (↑) $2T\cos\alpha = 2g$ M1 A1

$y = \frac{0.75}{\sin\alpha}$

Hooke's Law $T = \frac{49}{0.75}\left(\frac{0.75}{\sin\alpha} - 0.75\right)$ M1 A1

$= 49\left(\frac{1}{\sin\alpha} - 1\right)$

$\frac{9.8}{\cos\alpha} = 49\left(\frac{1}{\sin\alpha} - 1\right)$ Eliminating T M1

$\tan\alpha = 5(1 - \sin\alpha)$

$5 = \tan\alpha + 5\sin\alpha$ cso A1 6

[12]

1. The majority of candidates could complete, or nearly complete, part (a) successfully. The given answer did allow some candidates to correct their arithmetic errors. Those who reduced their 3 volumes to a ratio (8:19:27) before constructing the moments equation produced clearer and more straightforward solutions than those who worked with the original volume formulae. A few candidates remembered their GCSE work on the ratio of volumes of similar solids and stated directly that the ratio of the two hemispheres was 8:27.
Part (b) was done very quickly and easily by the most able candidates but proved difficult for the majority. Many candidates proceeded straight from Q3(a) to Q4. Of those who attempted part (b), there was a fairly even split between those who took moments about the vertical through *O* for the two weights (or mentally cancelled *g* and used masses) and those who found the coordinates of the centre of mass of the composite body. Some of the latter group found only one coordinate and others made an error in their calculations. Some of the former group produced a moments equation with trigonometrical ratios that cancelled. Since this made the given information that $\tan\theta = \frac{4}{5}$ redundant, they should have been alerted to their mistake.

2. This was a straightforward opening question but many candidates spoiled their solutions with careless errors. Most managed to use Pythagoras to calculate the length of the extended string. However some appeared to have misinterpreted the information given in the question and took the natural length as 12 m instead of 8 m. Hooke’s law was well known but problems arose as candidates were confused between the full string and half strings, with some thinking the tensions in these were different. Significant numbers gave the mass as their final answer instead of the weight as demanded. Part (b) gave rise to fewer problems. The formula for the elastic potential energy was known by all but a small number of candidates and many who had an incorrect extension in part (a) either completely recovered to gain full marks or gained 2 of the 3 marks by follow through.

3. Most candidates resolved horizontally and vertically in (a) and then found the tangent of their angle between the string and the horizontal or vertical before proceeding to obtain the tension. It was not uncommon to see solutions which started with the Pythagoras equation in line 3 of the mark scheme. This is a risky approach as errors in this equation cause many marks to be lost if the equation is not derived from the resolving equations. Hooke’s law and the formula for elastic potential energy were well known and frequently applied correctly in (b) to reach a correct answer. However, omission of some or all of the letters *m*, *g* and a in the final answer was fairly common.

4. Part (a) was well done. The only common error was considering the elastic potential energy in only one part of the string instead of in both parts. Most candidates realised that energy was involved and the few who attempted using Newton's Second Law almost all failed to consider a general point of the motion and so gained no credit. Nearly all candidates could start part (b) by resolving vertically and writing down some form of Hooke's Law. The manipulations required to obtain the required trigonometric relation, however, were demanding and even strong candidates often needed two or three attempts to complete this and the time spent on this was sometimes reflected in an inability to complete the paper. This was particular the case if candidates attempted to use or gain information by writing down an equation of energy. This leads to very complicated algebra and is not a practical method of solving questions of this type at this level. (Correctly applied it leads to a quartic not solvable by elementary methods.) For those who were successful in part (a), writing T in terms of , say, the angle made by each part of the string with the vertical proved the critical step. If they obtained $T = \frac{49}{0.75}\left(\frac{0.75}{\sin a} - 0.75\right)$, or its equivalent, the majority of candidates had the necessary trigonometric skills to complete the question.